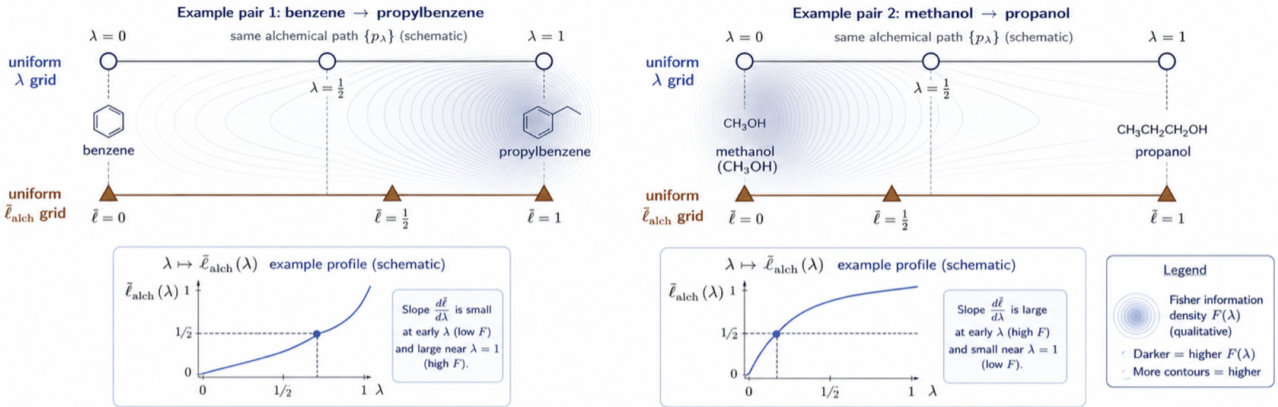
$$\ell_{\mathrm{alch}}(\lambda) = \int_0^{\lambda} \sqrt{F(\lambda')}\, d\lambda'$$

$$F(\lambda) = \beta^2 \operatorname{Var}_{p_\lambda}\left[\partial_\lambda U_\lambda\right]$$

Fisher–Rao arc length along the alchemical path; invariant to monotone reparameterizations of λ; equal $d\ell$ steps give equal leading-order KL steps: $\mathrm{KL}(p_\lambda \,\|\, p_{\lambda+d\lambda}) \approx \frac{1}{2}\, d\ell^2$.



Condition the shared flow on $s = \bar{\ell}_{\mathrm{alch}}$ instead of $s = \lambda$.